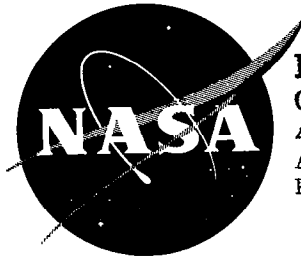


NASA TN D-46

NASA TN D-46

C.1



LOAN COPY: RETURN
Commander
Air Force Special Weapons Center
ATTN: Tech Info & Intel Office
Kirtland AFB, New Mexico



TECHNICAL NOTE

D-46

PLASMA ACCELERATION BY USE OF GUIDED MICROWAVES

By Robert V. Hess and Karlheinz Thom

Langley Research Center
Langley Field, Va.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON

December 1959



NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

TECHNICAL NOTE D-46

PLASMA ACCELERATION BY USE OF GUIDED MICROWAVES

By Robert V. Hess and Karlheinz Thom

SUMMARY

A method of plasma acceleration is explored which makes use of the radiation pressure of microwaves intensified in a resonant cavity as the driving pressure. When used to accelerate the plasma, the radiation can be compared to a gas which, however, is almost free of inertia and can expand with a velocity approaching the velocity of light.

With the assumption of a power input of 10^8 watts into a cavity, an input which lies in the range of possibilities because of the recent development of modern microwave generators, forces on the order of 1,000 newtons seem to be attainable for highly conducting thin plasma disks or rings. For such plasmas, velocities up to 10^6 m/sec and more are calculated. Although the energies that can be stored in the cavity do not yet quite measure up to acceleration systems where the energy is stored in condenser banks, the efficiency of interaction of the radiation with the plasma could be better than that of other systems, with the result that higher velocities may be obtainable for plasmas of small mass. Such plasmas may be useful for the study of certain aspects of thermonuclear fusion and propulsion.

INTRODUCTION

Within recent years great effort has been put into devising methods for electromagnetic acceleration of plasma for purposes of propulsion or thermonuclear fusion; in the latter case, the high kinetic energy is used as an intermediate step for the production of high temperatures through subsequent rapid deceleration of the plasma.

In this paper a method is explored for making use of the radiation pressure of guided microwaves for applying forces for acceleration of plasma. The success of such a method depends on the recent development of microwave generators of extreme power outputs which in the near future should reach 100 megawatts. The radiation pressures attainable with such high powers are further increased by storing the radiation by resonance in a finite volume, or cavity, which is bounded on one side by the

plasma. The input frequency of the standing wave in the resonant cavity is appropriately changed when the plasma moves, with the result that cavity dimensions are changed. Such changes in frequency are possible for modern microwave generators. When used to accelerate the plasma, the radiation can be compared to a gas which, however, is almost free of inertia and can expand with a velocity approaching the velocity of light.

A comparison of the present method with other systems for plasma acceleration and thermonuclear fusion would require much more detailed theoretical work combined with experimental studies. A few aspects are mentioned, however, which look promising. For example, in the rail type of accelerator, where the current passes from the walls through the plasma, the problem of contamination of the plasma with wall material arises sooner or later though ingenious ways have been devised to keep it small (ref. 1).

In the present method, the energy can be more or less focused in certain regions inside the waveguide so that this problem may be avoided. When a plasma is accelerated by the induction effects due to a moving magnetic field, produced either by motion of a wave in a delay line (ref. 2) or by a series of coils pulsed in sequence (ref. 3), the problem arises that the magnetic field runs over the plasma and tries to hook on to it in the manner of a synchronous or an asynchronous motor. In the present method the wave energy is confined behind the plasma and exerts a push.

Furthermore, high-frequency waves intersecting with a plasma will penetrate only a small amount (skin effect) provided the frequencies are not excessively high and the electron concentration of the ionized plasma is not too low. As a result, a good interaction of the waves with the plasma is accomplished even when the plasmas are made very thin and, thus, are of small mass. Because of the high repetition rate of pulses from a microwave generator, an almost continuous expulsion of plasma may be possible.

Methods for confinement of plasmas for thermonuclear fusion with radio-frequency oscillations were also considered, where the plasma is at the center or the axis of a cavity, but it was found that these methods had been developed in references 4 and 5 and in others. In such methods the losses due to skin effects at the wall are a drawback. The trend is thus to use oscillations for auxiliary confinement and dynamic stabilization in certain critical regions, with steady confinement doing the major task. (For example, see ref. 6.) The present method differs considerably in that the radiation pressure is used to perform work in short-time acceleration of the plasma. The skin-effect heating is thus beneficial to the accelerated plasma because it keeps it ionized and the losses at the metal end of the cavity are reduced because of the short time of acceleration. The losses in the sidewalls of the cavity can be kept very low. Approaches are also possible where both end walls of the cavity are made of plasma, thus keeping the losses to a minimum.

SYMBOLS

| | |
|----------------------|--|
| A | cross-sectional area |
| A',B',C' | constants |
| B | magnetic flux density |
| C _{refl} | reflection coefficient |
| c | velocity of light |
| d | diameter |
| E | electric field |
| F | force, newtons |
| f | input frequency, cycles/sec |
| G | guide constant, v_g/c |
| h | height of two-dimensional waveguide |
| $i = \sqrt{-1}$ | |
| k | absorption coefficient |
| l | number of half-period variations of radial electric field |
| m | mass, kg |
| n | index of refraction |
| P | power, w |
| Q | dimensionless quality factor of cavity, $\frac{2\pi \times (\text{Stored energy})}{\text{Energy lost per cycle of oscillation}} = \frac{\omega W}{P_{\text{loss}}}$ |
| q | unit charge |
| $R = \frac{2r}{x_0}$ | |
| R _M | magnetic Reynolds number |

4

| | |
|-------------------|--|
| r | radius of cylindrical waveguide |
| s | multiple of $\lambda/2$ |
| T | temperature, $^{\circ}\text{K}$ |
| $\text{TE}_{l,m}$ | mode of transverse-electric wave pattern in waveguide |
| t | time |
| V | volume |
| v_g | group velocity |
| W | energy |
| x | coordinate along waveguide axis |
| x_0 | length of cavity with fixed end disk, $s\lambda/2$ |
| δ | penetration depth of wave into conducting mass |
| ϵ_0 | dielectric constant of free space |
| η | efficiency |
| θ | angle defining position of radius vector of waveguide cross section |
| Λ | Debye shielding parameter |
| λ | wavelength |
| μ | magnetic permeability, $4\pi \times 10^{-7}$, $\frac{\text{weber}}{\text{amp-m}}$ |
| ν | collision frequency |
| σ | conductivity, mhos/m |
| $\chi_{l,m}$ | root of Bessel function |
| Ω | resistance |
| ω | circular frequency, $2\pi f$, radians/sec |

Subscripts:

| | |
|-----|--|
| co | cutoff |
| e | electron |
| g | conditions inside of waveguide |
| l | number of half-period variations of radial electric field |
| m | number of half-period variations of angular electric-field component with respect to radius of waveguide |
| max | maximum |
| o | cavity with fixed end disk |
| p | plasma |

Dots over a symbol indicate differentiation with respect to time.

ANALYSIS

Comparison of Forces Due to Wave Trains With and
Without Amplification by Resonance

The energy density or the radiation pressure of an electromagnetic wave train traveling along a tube of constant cross-sectional area A is given by the power transmitted P divided by A times the velocity of propagation. Thus,

$$\frac{dW}{dV} = \frac{P}{A\dot{x}} \quad (1)$$

where

$$dV = A dx$$

and

$$dW = P dt$$

The force which the wave train exerts on a disk of conducting material of cross-sectional area A then will be

$$F = \frac{P}{A\dot{x}} A = P \frac{dt}{dx} \quad (2)$$

In the case of a constant velocity of wave propagation, equation (2) simplifies to

$$F = \frac{Pt}{x} = \frac{W}{x} \quad (3)$$

In the case of unrestricted propagation in the direction of the axis of the waveguide, the energy Pt is distributed over a volume which is A times the distance traveled by the wave during the time t . Thus, the force due to absorption of a single wave train is

$$F = \frac{Pt}{ct} = \frac{P}{c} \quad (4a)$$

For a reflected wave the force would be doubled; that is,

$$F = \frac{2P}{c} \quad (4b)$$

If it is assumed that the power transmitted P is 10^8 watts, the forces due to absorption and reflection of a single wave train are, respectively,

$$F_{\text{abs}} = \frac{1}{3} \text{ newton}$$

and

$$F_{\text{refl}} = \frac{2}{3} \text{ newton}$$

It is evident that if the wave train is transmitted into a finite volume, where it gets a chance to be reflected back and forth, a resonant

intensification of the amplitudes of the electromagnetic waves can take place, and with it there is intensification of radiation pressure. (See fig. 1.) In the case of intensification by resonance, the energy Pt is distributed over a restricted volume Ax_0 , where x_0 is the cavity length and A is the cross-sectional area of the cavity. Thus, the force from equation (3) becomes

$$F = \frac{W}{x_0} = \frac{Pt}{x_0} \quad (4c)$$

It should be briefly pointed out that equation (4c) can be obtained by considering waves that are many times reflected and which in detail produce the resonant intensification of the force. For a single reflection the force is doubled in accordance with equation (4b). Now, in the energetic approach (eq. (4a)) the force is obtained from the energy in the complete cavity. In order to find agreement with this approach, note that a wave train has to travel through twice the length x_0 of the cavity for the whole cavity to realize the doubling of the force. The doubled force thus has to be divided by $2x_0$, which results in the simple expression given by equation (4c). For orientation, a numerical example is given. If it is assumed that the cavity length is 1 meter and the duration of the microwave pulse is 10^{-5} seconds, then

$$F = 10^3 \text{ newtons}$$

and the stored energy is

$$W = 1,000 \text{ joules}$$

With longer pulses which are probably attainable by phasing of a sequence of pulses (as used in microwave-particle accelerators), even higher values of W can be obtained through addition of energy. These high values reveal the possibilities of energy storage. However, limitations by energy losses due to the reflections of the wave train at the boundaries of the cavity must be expected.

In regard to the power of 10^8 watts, high-power microwave generators of 17 megawatts were available in 1954 (ref. 7). With such generators, powers of the order of 100 megawatts can be attained by permitting several generators to be coupled to a cavity. According to the latest information, powers of 100 megawatts are becoming a practical possibility.

Broad Outline of Intensification Including Losses in Cavity Walls

The rate of energy storage in a cavity is expressed as the difference between power input and power loss. Thus,

$$\frac{dW}{dt} = P - P_{\text{loss}} \quad (5)$$

where P is the power transmitted in the longitudinal direction of the cavity. The ratio of stored energy to the energy dissipated is given by the quality factor Q in waveguide theory. Thus,

$$Q = \omega_g \frac{W}{P_{\text{loss}}} \quad (6)$$

where $\omega = 2\pi f$ is the circular frequency. The quality factor Q depends on the frequency, geometry, and material of the cavity as discussed in a subsequent section. Substitution of equation (6) into equation (5) gives

$$\left. \begin{aligned} dW &= -\frac{\omega_g}{Q} \left(W - P \frac{Q}{\omega_g} \right) dt \\ \frac{dW}{W - P \frac{Q}{\omega_g}} &= -\frac{\omega_g}{Q} dt \\ W - P \frac{Q}{\omega_g} &= A e^{-\left(\omega_g/Q\right)t} \end{aligned} \right\} \quad (7)$$

Since $W = 0$ when $t = 0$,

$$A = -P \frac{Q}{\omega_g}$$

and

$$W = P \frac{Q}{\omega_g} \left(1 - e^{-(\omega_g/Q)t} \right) \quad (8a)$$

Thus, the maximum energy which can be stored in a cavity for $t \rightarrow \infty$ is

$$W_{\max} = P \frac{Q}{\omega_g} \quad (8b)$$

This maximum value for the energy evidently could have been obtained directly from equation (5) by setting $dW/dt = 0$. Substitution of the resulting equality of P and P_{loss} into equation (6), which defines Q , yields equation (8b).

Equations (8a) and (8b) show that although the maximum storable energy will increase with Q , the pulse required to attain these high energies would have to increase, since Q appears also in the expression $e^{-(\omega_g/Q)t}$. On the other hand, when high frequencies are used, the maximum stored energy will be obtained faster; however, this value will be comparatively reduced since the frequency appears also in the denominator of the expression for W_{\max} . Note also that for most practical situations encountered in the waveguide accelerator, ω_g/Q will be considerably larger than unity; thus, even for short pulses the expression $e^{-(\omega_g/Q)t}$ will be sufficiently large, and as a result the deviation from W_{\max} will be small. This situation is also conveniently expressed in terms of efficiency η of energy storage as follows:

$$\eta = \frac{\text{Stored energy}}{\text{Total energy input}} = \frac{P \frac{Q}{\omega_g} \left(1 - e^{-(\omega_g/Q)t} \right)}{Pt} = \frac{Q}{\omega_g t} \left(1 - e^{-(\omega_g/Q)t} \right) \quad (9)$$

Waveguide Theory Leading Up to Numerical Values of Q

The guiding of waves in a waveguide is obtained by reflection of the waves in the waveguide from the sidewalls of the waveguide (fig. 2). Because of the zig-zag progress of the traveling wave, the rate of progress of the pulse along the axis of the waveguide must be slower than that of the input wave traveling at light velocity c . This slower

rate of progress of the energy in the longitudinal direction is the so-called group velocity v_g (see, for example, pp. 193-227 of ref. 8) which is expressed as

$$\frac{v_g}{c} = G = \frac{\lambda}{\lambda_g} = \sqrt{1 - \left(\frac{\lambda}{\lambda_{co}}\right)^2} = \sqrt{1 - \left(\frac{f_{co}}{f}\right)^2} \quad \left(\lambda = \frac{c}{f}\right) \quad (10)$$

and

$$\frac{f_g}{f} = \frac{v_g}{\lambda_g} \frac{\lambda}{c} = \frac{v_g^2}{c^2}$$

where v_g/c is designated as the guide constant G . The subscript co refers to the cutoff wavelength; the quantities with subscript g refer to the wavelength of the transmission along the guide axis; the simple expression for λ_{co} in figure 2 is for the schematic two-dimensional case. The cutoff wavelength λ_{co} signifies that for an input wavelength ($\lambda > \lambda_{co}$), the input waves move normally to the walls of the guide and can no longer be transmitted. It should be noted that the power transmitted along the guide axis P_g is equal to P ; however, since the value of G is not far from 1 for the high frequencies involved, $P_g \approx P$; for purposes of analysis, the transmission is assumed to be approximately one dimensional.

The intersection of reflected waves can produce a great variety of wave patterns or modes to be propagated along the waveguide. Among the various possible modes the $TE_{0,m}$ modes in cylindrical tubes are chosen because they have the important property that the electric-field lines are concentric circles. The charged particles making up the plasma are thus not thrown against the wall, a result which avoids cooling of the plasma and contamination with the wall material. The $TE_{0,m}$ mode has also the unusual property that the losses in the sidewall of the waveguide decrease with increasing frequency. (In ref. 9 the $TE_{0,m}$ mode is the same as the H_{0m} mode.) This can be shown to be related to the fact that no electric-field lines terminate at the walls.

The meaning of the subscript m can be best understood from a study of the two wave patterns in figure 34 of reference 10 where the $TE_{0,1}$ and $TE_{0,2}$ modes are shown. In defining $TE_{0,m}$, the subscript 0

refers to the number l of full-period variations of E_r (radial electric field) with respect to θ , and the subscript m refers to the number of half-period variations of E_θ with respect to radius r . Figure 34 of reference 10 is repeated here for convenience in the form of figures 3 and 4. The choice of the subscript m for the $TE_{0,m}$ mode is indicated by the ratio of the cutoff wavelength λ_{co} to the radius of the cylindrical tube. As shown in table 1 on page 639 of reference 10, for $m = 1$ and $m = 2$, $\lambda_{co} = 1.64r$ and $0.89r$, respectively.

The choice of the radius r of the tube is also influenced by the fact that the mass of the plasma to be accelerated should not be too large. By using a radius of 6 centimeters, $\lambda_{co} = 9.84$ centimeters for $m = 1$ and $\lambda_{co} = 5.34$ centimeters for $m = 2$. Since the production of very high powers becomes difficult for extremely small wavelengths, a cutoff wavelength of $\lambda_{co} = 5.34$ centimeters would seem rather small for practical purposes, since the input wavelength has to be smaller than the cutoff wavelength λ_{co} . The possibility of using $TE_{0,2}$ or $TE_{0,3}$ modes should, however, be kept in mind for the cases where a more even distribution of the magnetic field and the corresponding magnetic pressure is desired. (See figs. 3 and 4.)

The wavelength in the guide is given from equation (10) by

$$\lambda_g = \frac{c}{fG}$$

The guide constant G is given for the $TE_{0,1}$ mode; for example, in reference 8, page 217,

$$\left. \begin{aligned} G &= \sqrt{1 - \frac{f_{co}^2}{f^2}} = \sqrt{1 - \frac{0.371c^2}{f^2 r^2}} = 0.952 = \frac{v_g}{c} \\ \lambda_g &= \frac{\lambda}{G} = 3.16 \text{ centimeters} \end{aligned} \right\} \quad (11)$$

For the present exploratory calculations an input frequency f of 10^{10} cycles/sec is chosen. This is only a sample value which is more or less adjusted to a particular waveguide cross section. The

corresponding input wavelength $\lambda = c/f = 3$ centimeters is a little small for achievement of the extreme power outputs of the order of 100 megawatts. However, for a somewhat larger waveguide cross section, wavelengths of the order of 10 centimeters would be usable, for which extreme powers are rapidly becoming available. In this connection, it should be noted that the $TE_{0,m}$ mode has its largest fixed strengths in a ring-shaped region away from the wall. (It is sometimes called the doughnut mode.) It becomes possible to save plasma weight by exerting pressure on plasma rings instead of filling out a large part of the cross section of the waveguide. The radiation pressure on the ring would, of course, no longer correspond exactly to the average energy density of the radiation based on quasi-one-dimensional considerations. Rather, a detailed consideration of the field configuration would be required. In this connection, it should be pointed out that the equivalence of magnetic pressure to energy density is only strictly true provided the magnetic lines of force are straight and parallel. (For example, see p. 24 of ref. 11.)

The Quality Factor Q and Its Variation With Plasma Properties

Formal development.— The dependence of Q on the geometry of the cavity, conductivity of the cavity walls, and input wavelength is given on page 300 of reference 12 for any $TE_{l,m,s}$ mode as

$$Q \frac{\delta}{\lambda} = \frac{\left[1 - \left(\frac{l}{x_{l,m}} \right)^2 \right] \left(x_{l,m}^2 + p^2 R^2 \right)^{3/2}}{2\pi \left[x_{l,m}^2 + p^2 R^3 + (1 - R) \left(\frac{pRl}{x_{l,m}} \right)^2 \right]} \quad (12)$$

where, in reference 12,

$$\delta = \sqrt{\frac{2}{\mu\sigma\omega}} = \sqrt{\frac{\lambda}{\mu\sigma\pi c}}$$

with δ being the depth of penetration of the current due to the skin effect for metals referred to the input wavelength λ , $p = \frac{8\pi}{2}$, and

$R = \frac{2r}{x_0}$. The quantities $x_{l,m}$ refer to the roots of Bessel functions (ref. 12, p. 299). For the $TE_{0,m,s}$ modes used here, l is zero.

The subscript s , which has been added since the waveguide is closed off to become a cavity, refers to the number of half-period variations

in the direction of the waveguide axis. In other words, the length of the cavity is given in multiples s of $\lambda_g/2$.

The effect on Q of the variation in cavity length at constant waveguide radius is now discussed. By using the fact that the length of the resonant cavity has to be a multiple of the wavelength in the cavity $x_0 = \frac{s}{2} \lambda_g$, Q becomes for the $TE_{0,m,s}$ modes

$$\left. \begin{aligned} Q &= \frac{\lambda}{\delta} \frac{\left(x_{0,m}^2 + 4\pi^2 \frac{r^2}{\lambda_g^2} \right)^{3/2}}{2\pi \left(x_{0,m}^2 + \frac{16\pi^2}{s} \frac{r^3}{\lambda_g^3} \right)} \\ \lim_{s \rightarrow \infty} Q &= \frac{\lambda \left(x_{0,m}^2 + 4\pi^2 \frac{r^2}{\lambda_g^2} \right)^{3/2}}{\delta 2\pi x_{0,m}^2} \end{aligned} \right\} \quad (13)$$

Equation (13) indicates that for a constant guide radius r and a given wavelength λ , Q increases with increasing length x_0 , that is, with multiples s of the wavelength $\lambda_g/2$. The smallest Q is obtained for the minimum length $x_0 = \frac{\lambda_g}{2}$, or $s = 1$. When s approaches ∞ , Q approaches a limiting value.

The introduction of higher multiples s of the wavelength $\lambda_g/2$ indicates the introduction of higher modes for the $TE_{0,m,s}$ pattern. The introduction of such higher modes is known to increase the quality factor Q . The reason is, in essence, that for each wavelength the number of boundaries, where the losses occur, is reduced. An increase in Q can be also obtained by increasing the cross section of the waveguide, a condition which offers the possibility of introducing higher modes in the direction of the waveguide radius. Even if no higher mode is introduced, the enlarging of the cross section would increase Q by virtue of the fact that the larger cross section removes the guide frequency further from cutoff frequency. A way of reducing the losses at the ends of the cavity is to reduce the frequency. Since, in this manner, the cutoff frequency is approached, the losses in the sidewalls will increase, unless the cross section of the waveguide is made proportionately bigger. The optimum choice of frequency will, thus, be based on a compromise. It should be reemphasized that for the $TE_{0,l,s}$ mode

the maximum intensity of the electric field occurs in a ring-shaped region, with the result that the plasma has to be only of ring shape and as a consequence is at reduced weight. This is especially important when the larger waveguide cross sections are considered which offer the possibility of increased efficiency.

Interaction of electromagnetic radiation with a plasma and the effect on Q . Equation (13) indicates that the quality factor Q is inversely proportional to δ , the depth of penetration or skin depth of the electromagnetic radiation into the cavity wall. For purposes of orientation, δ is first expressed by the approximate formula used for metals for microwave frequencies, without discussing its exactness for plasmas, that is,

$$\delta = \sqrt{\frac{2}{\mu\sigma\omega}}$$

Since Q is also about proportional to λ or inversely proportional to ω , it follows that approximately

$$Q \propto \sqrt{\frac{\sigma}{\omega}}$$

This is not surprising if it is remembered that Q is inversely proportional to losses and that the resistance Ω is of the dimension

$$\Omega = \frac{1}{\sigma\delta} \propto \sqrt{\frac{\omega}{\sigma}}$$

It is thus indicated that in order to have a high value of Q , the electric conductivity should be made high and the frequency low. However, the severity of this effect is somewhat diminished because ω and σ appear under the square-root sign.

The use of high frequency has, however, certain important advantages in promoting efficient interaction of the electromagnetic radiation with the plasma, the interaction which indirectly has also a beneficial influence on the quality factor Q . As previously discussed, the use of high frequencies is desirable for keeping the waveguide to reasonable size; but, of course, this in itself is not sufficient justification for the waveguide method.

The advantages are conveniently expressed in terms of the magnetic Reynolds number R_M which gives the efficiency of interaction between

a magnetic field and a plasma. Limitations to this approach are subsequently discussed. The nondimensional parameter R_M may be expressed in the various forms

$$R_M = \mu \sigma v L$$

$$R_M = \frac{\mu \sigma L^2}{t}$$

$$R_M = \mu \sigma L^2 \omega$$

$$R_M = \frac{L^2}{1/\mu \sigma \omega}$$

$$R_M = \frac{L^2}{\delta^2}$$

where L denotes length and v denotes velocity. It is indicated that in order to improve the interaction of plasma and the magnetic field for a given value of σ and μ , it is desirable to use short interaction times or high frequencies. The large magnetic Reynolds number thus attained can also be interpreted to mean that the magnetic field cannot penetrate deeply into the plasma or that the skin depth δ is small. This is important in any method of plasma acceleration because it permits the use of thin and thus small weight plasmas.

From the previous discussion, the high frequency (within certain limits subsequently discussed) is thus seen to be a necessary condition for efficient interaction, but it is not sufficient for creating high values of Q which are approached as follows: An increase in frequency ω decreases the penetration depth δ in the plasma and with it increases the resistance Ω and the heating. Because of the heating, the conductivity σ is increased which in turn increases the quality factor Q . The frequency will have to be judiciously chosen so that it optimizes these conditions together with the waveguide requirements. It must be also taken into consideration that the power output of transmitters is reduced with increase in frequency or the corresponding reduction in wavelength.

So far, the plasma has been treated like a metal. It must be emphasized, however, that at the high frequencies under consideration

the plasma does not always act like a metal but may behave like a dielectric or a combination of both. It will be shown that this behavior of the plasma can have certain beneficial effects on the quality factor Q . The various cases of dielectric or metal characteristics can be expressed with comparative simplicity for an electromagnetic wave impinging perpendicularly on the plasma. The variation of the plasma properties with frequency is most conveniently expressed in terms of the complex dielectric constant or its square root, the index of refraction.

A wave propagating in a medium with the index of refraction n and the absorption coefficient k has the complex index of refraction (see, for example, eq. (3.2) on p. 112 in ref. 13) which is expressed as

$$n - ik = \sqrt{1 + \frac{\omega_p^2}{-\omega^2 + i\omega\nu}} \quad (14)$$

where ν is the collision frequency. The sum terms in reference 13 are neglected here since they refer to bound electrons which are not considered for the plasma case.

The frequency ω_p is the so-called plasma frequency and is given by

$$\omega_p = 2\pi \sqrt{\frac{10^6 N_e q^2}{m_e \epsilon_0}} = 2\pi \times 8.97 \times 10^3 \sqrt{N_e} \quad (15)$$

where N_e represents the number of electrons per cubic centimeter.

Now, it is known (ref. 13) that, for a metal, n and k both approach

$$n = k = \sqrt{\frac{\sigma}{2\omega\epsilon_0}} \quad (16)$$

The depth or distance of decay in amplitude to $1/e$ (where e is the base of the natural logarithm) is

$$\delta = \frac{1}{k \frac{\omega}{c}} = \sqrt{\frac{2\omega\epsilon_0}{\sigma}} \frac{1}{\omega/c} = \sqrt{\frac{2}{\mu\sigma\omega}} \quad (17)$$

where δ is the well-known skin depth previously used.

It can be readily shown that the plasma acts like a metal under the somewhat contradictory conditions

$$\left. \begin{aligned} \frac{\nu}{\omega} &> 1 \\ \frac{\omega \nu}{\omega_p^2} &= \frac{\nu}{\omega} \frac{\omega^2}{\omega_p^2} < 1 \end{aligned} \right\} \quad (18)$$

If the expression for ω_p in equation (15) is squared, and if the direct-current conductivity σ is given by

$$\sigma = \frac{N_e q^2}{m_e \nu} \quad (19)$$

the result is

$$\frac{\omega \nu}{\omega_p^2} = \frac{\epsilon_0 \omega}{\sigma}$$

Substitution of this relation into equation (14) and using the inequalities in equation (18) reduce equation (14) to

$$n - ik = \sqrt{\frac{1}{i} \frac{\sigma}{\epsilon_0 \omega}} = (1 - i) \sqrt{\frac{\sigma}{2\epsilon_0 \omega}} \quad (20)$$

which agrees with the conditions for n and k stated in equation (16) as applied to a metal.

For a given electron concentration N_e or plasma frequency ω_p , the plasma will thus act as a metal provided its direct-current conductivity (eq. (19)) is low with the added condition that the input frequency lies sufficiently far below the plasma frequency. The reason for a metal behaving as it does is that its electron concentration and, thus, its plasma frequency are extremely high.

On the other hand, if the collision frequency ν is small compared with ω in equation (18) and if the direct-current conductivity is large, the plasma acts like a dielectric with the complex index of refraction

$$n - ik = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \quad (21)$$

For $\omega > \omega_p$, a real index of refraction exists:

$$n = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

When $\omega < \omega_p$, however, k which was actually set up as a decay or absorption parameter becomes real:

$$k = \sqrt{\frac{\omega_p^2}{\omega^2} - 1}$$

The distance required for the amplitude to decay to $1/e$ is

$$\delta = \frac{1}{k \frac{\omega}{c}} = \frac{c}{\omega_p} \frac{1}{\sqrt{1 - \frac{\omega^2}{\omega_p^2}}} \quad (22)$$

The finite penetration of the wave is here not due to heat losses brought about by collisions but represents the exponential "tail" due to total reflection from the now dielectric plasma. (See ref. 11.) If the frequency is smaller than the plasma frequency, finite penetration depth results evidently from purely dielectric processes.

In the limiting case where the input frequency ω is negligible compared with ω_p (the plasma frequency), equation (22) degenerates to the form

$$\delta = \frac{c}{\omega_p}$$

Since the frequency ω is now negligible, this penetration depth must be the result of a pure direct-current phenomenon, where the effects of collisions have become negligible and, thus, the direct-current conductivity is infinite. The finite penetration of the wave can, however, no longer be explained in steady-state terms. An interesting independent derivation of this limiting case of zero frequency is given in reference 14 and is based on time-dependent considerations which are, however, not of the periodic nature used here. It should be recalled here that the customary derivation of the direct-current conductivity which is, in contrast, based on the attainment of a terminal constant electron velocity is produced by friction due to collisions.

The interaction of the wave with the plasma so far has been based on the assumption that the electric fields are weak. As is known from the study of runaway electrons for strong fields (for example, see ref. 15), the customary concept of direct-current conductivity (based on reaching terminal constant electron velocities due to collisional "friction") may no longer be a good approximation and time-dependent concepts have to be introduced. This thought has, in essence, already been expressed in explaining the limiting case of infinite direct-current conductivity. The theory of runaway electrons merely indicates that for strong fields the time-dependent effects become important even before this limiting case is reached. This is also true of problems treated here where comparatively high frequencies are involved, although this behavior is not due to the use of strong fields. Thus, it appears that for high powers and strong oscillating fields, the time-dependent effects will have even greater importance. The runaway of the electrons will, of course, be impeded because of oscillation of the fields.

It should be pointed out that in order for the plasma to act like a dielectric, its boundary has to be comparatively sharp. (The distance over which the changes in dielectric constant occurs must be considerably smaller than the wavelength.) Since the plasma is pushed by the radiation pressure, it should not be difficult to maintain such a sharp boundary. The stability of the plasma configuration would also seem good since the high-frequency oscillations offer a possibility of dynamic stabilization. The radiation pressure is essentially transmitted to the lighter electrons. They, in turn, transmit it to the ions by collisions or, for low-collision frequencies, by space-charge effects.

For a more detailed evaluation, the possibility of growing coherent plasma oscillations with Landau damping (ref. 11) and nonlinear properties would also have to be considered, including possible deformations of the plasma and the production of higher harmonics; since the problem of interaction of high-power radiation with a plasma is rather new, further theoretical and experimental studies are necessary to check the aforementioned relations.

Since the quality factor Q is inversely proportional to the losses, a brief discussion of them is now given. The power-loss coefficient (for the plasma thickness larger than δ when the power transmission through the rear boundary is negligible) is expressed as (p. 125 in ref. 13)

$$1 - C_{\text{refl}} = 1 - \frac{(1 - n)^2 + k^2}{(1 + n)^2 + k^2} \quad (23)$$

where C_{refl} is the coefficient of reflected power. The unity in $1 \pm n$ refers to the index of refraction of the region outside of the plasma.

From the severe conditions of inequalities in equation (18) required of the plasma to act like a metal, it is to be expected that for the increasingly high direct-current conductivities expected because of the high-frequency plasma heating, the plasma will tend to act increasingly more like a dielectric with total reflection of the impinging radiation. As a consequence, the losses for the plasma should become smaller than those of the metal with the same direct-current conductivity, and as a result the quality factor Q should become higher. Calculations for various direct-current conductivities and waveguide frequencies agree with these effects. The losses were also calculated by an approximate relation given in equation (2) of reference 4 for the power input per unit area due to ohmic heating

$$P = \frac{1}{\sigma \mu \delta} \frac{B^2}{2\mu} \quad (24)$$

by using for δ the penetration depth $\frac{1}{\frac{\omega}{c}/k}$ of combined metallic and dielectric origin; the expression $B^2/2\mu$ is the energy density due to radiation at the plasma surface. A good check on the order of magnitude of the more involved losses from equation (23) was obtained. Equation (24) brings out the fact that the loss reduction in the plasma compared with that of the metal corresponds to a larger penetration depth as the dielectric total reflection is approached with increasingly larger direct-current conductivity. It should be emphasized, however, that even for the comparatively high frequencies of $\omega = 2\pi \times 10^{10}$ radians/sec chosen in this paper, the penetration depth for total dielectric reflection in equation (22) would still be small. If, for example, an electron concentration of $N_e = 10^{14}$ per cubic centimeter is chosen, the plasma frequency ω_p in equation (15) would be

$$\omega_p = 2\pi \times 8.97 \times 10^3 \sqrt{N_e} = 2\pi \times 8.97 \times 10^{10} = 5.63 \times 10^{11}$$

The resulting ratio is

$$\frac{\omega^2}{\omega_p^2} = \frac{(2\pi \times 10^{10})^2}{(2\pi \times 8.97 \times 10^{10})^2} = \frac{1}{80.46}$$

and δ becomes

$$\delta = \frac{3 \times 10^8}{5.63 \times 10^{11}} \frac{1}{\sqrt{1 - \frac{1}{80.46}}} = 0.54 \text{ millimeter}$$

This value of δ is still very small as a lower limit for practical plasma thicknesses. The metallic penetration depth given in equation (17) would, of course, be much smaller. For example, for silver with $\sigma = 6.13 \times 10^7$ mhos/m and $\omega = 2\pi \times 10^{10}$ radians/sec, the penetration depth is

$$\delta_{\text{metal}} = \sqrt{\frac{2}{\mu\sigma\omega}} = 6.4 \times 10^{-7} \text{ meter} = 6.4 \times 10^{-4} \text{ millimeter}$$

These approximate considerations indicate that a treatment of the quality factor Q of the plasma on the assumption that the plasma acts as a metal should yield conservative values, whereby the extremely small penetration depth existing for interaction of waves with metals can no longer be maintained. This condition seems, however, of no serious practical consequence in the choice of plasma dimensions, since the penetration depth for the plasma is still small.

It should be noted also that the plasma acts as only one wall of the cavity (fig. 1) and that, thus, the value of Q of the complete cavity will be due to a combination of wave interactions with metal and plasma regions. For a plasma of lower than metallic reflection, the value of Q based on the plasma alone would thus seem a conservative one. If the interesting possibility were to arise that the plasma Q is higher than that of the metal, naturally the metallic Q would have

to be taken. Of course, further theoretical and experimental studies of the nonlinear interaction effects are necessary before passing final judgment. Since the plasma proves to be a very good reflector at high temperatures, special designs of plasma accelerators have also been considered where both end walls of the cavity are made of plasmas with arrangements enabling alternate replacement of the plasmas after acceleration. In this manner, the power generally lost due to skin effect could be put to use.

It would seem perhaps worthwhile to point out that for a quality factor Q that does not have excessively large values (that is, the system is not too sharply tuned), its value will not be too sensitive to changes in configuration. Also, as previously stated, the time required to reach maximum-energy storage in the cavity will be shorter and with it the pulse required will be shorter. A nonexcessively high value of Q may, thus, offer fewer disadvantages than those expected on first sight.

The effect of coupling from the transmitter into the cavity must, of course, also be considered. With modern means of coupling, the losses should not be too large, especially if Q is not excessively high. The problem of breakdown at the windows of the cavity for extreme-power microwaves would have to be given thorough experimental attention.

Calculation of Q . - Since only few studies of the interaction of high-power radiation with plasmas have been performed, only approximate values of Q can be given here. Since it was shown in the previous section that the use of metallic or direct-current values for the plasma Q should give conservative estimates, it was decided to use such values for the sake of simplicity. The values of Q to be calculated are based on two representative plasma direct-current conductivities, whereby for each conductivity the variation of Q with changes in cavity length is taken into account.

The representative values for the direct-current plasma conductivities are now given. Assume a hydrogen plasma that has been preheated, say, by radiation itself or by some induction process which can put heat in a ring region, to a temperature of $10,000^\circ \text{K}$ to $12,000^\circ \text{K}$ where expressions for full ionization become applicable. The direct-current conductivity is then obtained from the distance-encounter expression (eq. (5-37) in ref. 11) as

$$\sigma = 1.53 \times 10^{-2} \frac{T^{3/2}}{\log_e \Lambda} \text{ mhos/m}$$

Assume that the ionized plasma contains approximately 10^{14} particles; these particles are equally divided into ions and electrons. Now, for an electron density of approximately $\frac{1}{2} \times 10^{14}$, the value of $\log_e \Lambda$ according to table 5.1 (p.73) in reference 11 should be about 7. A conductivity is thus obtained which is larger than 10^3 mhos/m; however, 10^3 is taken as one representative value. Another value for the conductivity chosen is the high value for pure silver where $\sigma = 6.13 \times 10^7$ mhos/m.

The values of Q for $\sigma = 10^3$ mhos/m together with the previously chosen and calculated values for frequency and cavity radius are obtained by substitution of σ into equation (12) and are given in table I(a). The cavity length is expressed in terms of multiples s of $\lambda_g/2$, where λ_g is the wavelength inside the guide which has been previously determined and is 3.16 centimeters. It is of interest to note that Q varies from the smallest value of 55.74 at $s = 1$ to its largest value of 4.29×10^3 as $s \rightarrow \infty$. However, even at finite large values of s , Q is of the order of 10^3 . The values for Q are conservative since only one wall of the metallic cavity has the comparatively low plasma conductivity of $\sigma = 10^3$ mhos/m. Similar calculations using the much higher direct-current conductivity of silver, $\sigma = 6.13 \times 10^7$ mhos/m, result in the values of Q given in table I(b).

A brief investigation of the effect of superconductivity on the metal walls was also made. Since, as is well known, the superconducting state tends to break down for certain magnetic-field strengths (related to the currents), the superconductivity should most likely find its greatest use in the sidewalls of the waveguide. The reason is that for the $TE_{0,m}$ modes used here the currents and the associated losses in the sidewalls are comparatively small. The effect of high-frequency currents on superconduction would have to be studied in more detail.

In order to determine the value of Q for the sidewalls that are superconducting (the losses due to them are zero), the power losses P_{loss} in equation (6) and indirectly contained in equation (12) for Q would have to be split into those due to sidewalls and those due to the end disks. Thus,

$$Q = \frac{\omega W}{P_{\text{loss,side}} + P_{\text{loss,end}}} \quad (25)$$

In these exploratory considerations an exact derivation of the split for the general value of Q in equation (12) is avoided. The approach adopted is rather to use Q for $s = 1$ as the starting point for the superconducting sidewalls. The reason is that for $s = 1$ the length of the cavity is $\lambda_g/2$ or only 1.58 centimeters (see eq. (11)); the radius, on the other hand, which is the same for all values of s , has the value of 6 centimeters. It stands to reason that in this case with $s = 1$ the losses will be predominantly due to the end disks. As a consequence, the value of Q for $s = 1$ will give a conservative estimate of the value of Q for $s = 1$ with superconducting sidewalls. When the losses in the sidewalls can be neglected, the denominator of equation (25) remains constant since the losses in the end disks remain the same for a given frequency. On the other hand, the stored energy in the numerator of equation (25) is proportional to the volume of the cavity, and for a given cavity radius the numerator is proportional to the length of the cavity. Since the value of Q for the superconducting sidewalls (Q_{super}) was chosen to be equivalent to the value of Q for the ordinary sidewalls with $s = 1$, the proportionality to the cavity length enters through multiplication of $(Q_{\text{super}})_{s=1}$ by s . The variation of Q_{super} with s is given for $\sigma = 10^3$ mhos/m in table I(a) and for $\sigma = 6.13 \times 10^7$ mhos/m in table I(b). Since the identification of Q_{super} for $s = 1$ with the ordinary case for $s = 1$ is somewhat conservative, Q_{super} for $s = 1$ was rounded off to a slightly higher value. Specifically, Q for $\sigma = 6.13 \times 10^7$ mhos/m was rounded off to 1.4×10^4 for Q_{super} , and the value of Q_{super} for $\sigma = 10^3$ mhos/m was correspondingly obtained by multiplying Q_{super} by the square root of the ratio of the two conductivities.

L
4
5
1

The Force on the End Disks of the Cavity

Since the force on the end disks of the cavity is W/x_0 , it can be written with the use of equations (8) as

$$F = \frac{P}{x_0} \frac{Q}{\omega_g} \left(1 - e^{-(\omega_g/Q)t} \right) \quad (26)$$

With $x_0 = s \frac{\lambda_g}{2}$, $\omega_g = 2\pi f_g$, and $\lambda_g = \frac{c}{f_g}$, the maximum force is

$$F_{\text{max}} = \frac{P}{x_0} \frac{Q}{\omega_g} = \frac{PQ}{\pi f \lambda_g s} \quad (27)$$

For $P = 10^8$ watts, $f = 10^{10}$ cycles/sec, and $\lambda_g = 3.16 \times 10^{-2}$ meter, equation (27) can be reduced to

$$F_{\max} = \frac{Q}{3.16\pi} \frac{1}{s} \quad (28)$$

In spite of the simple expression for F_{\max} for computational purposes, it is of interest to give a general analytical expression for F_{\max} . For losses that occur in the sidewalls, Q is of the form

$$Q = \frac{A'}{B' + \frac{C'}{s}} \quad (29)$$

where A' , B' , and C' are constants. The maximum force F_{\max} is of the form

$$F_{\max} = \frac{A_1'}{B_1's + C'} \quad (30)$$

In other words, F_{\max} decreases with cavity length (the dimension of the force is energy over length) in contrast with Q which shows an increase.

On the other hand, when the sidewalls are superconducting, Q is proportional to the cavity length or s , the multiple of $\lambda/2$; and, thus, F_{\max} from equation (28) remains constant with varying cavity length. The variation of F_{\max} with s is given for $\sigma = 10^3$ mhos/m in figure 5; the constant F_{\max} for superconducting sidewalls is $F_{\max} = 5.69$ newtons. The variation of F_{\max} with s for $\sigma = 6.13 \times 10^7$ mhos/m is given in figure 6. The constant F_{\max} for superconducting sidewalls is $F_{\max} = 1.41 \times 10^3$ newtons.

At this point an important observation seems in order concerning the benefit of storing the energy by resonance rather than permitting the radiation merely to impinge on the plasma without resonant intensification. For the nonresonant case the force on the plasma is $1/3$ or $2/3$ newton, depending on whether the plasma completely absorbs the radiation or reflects it completely. (See eqs. (4b) and (4c) and the

following discussion.) Calculations of the reflection coefficient in equation (23) from the plasma data indicate that even for $\sigma = 10^3$ mhos/m the plasma will still offer some reflection; consequently, this force should be over 1/3 newton. For the resonant case, however, with $\sigma = 10^3$ mhos/m, the relatively low conductivity, a conservative estimate results on the average (fig. 5) in a force of at least 10 times larger. For the higher conductivity of $\sigma = 6.13 \times 10^7$ mhos/m, the increase in force is, of course, considerable, being on the average 1,000 times higher. It must be recognized, however, that the force for the nonresonant case remains about constant with changing cavity length because the wave train which exerts pressure on the plasma traverses the path to the plasma only once; whereas, in the resonant cases the repeated reflections cause added losses (except for superconducting sidewalls) and thereby lower the force. It will be shown subsequently that, for the acceleration of very small masses, the success of the waveguide accelerator does not depend on the effect of resonance; but, of course, it is greatly benefited by it.

Equations for Plasma Acceleration

The energy balance of a resonant cavity with one end disk permitted to accelerate in the direction of the guide axis (fig. 1) is

$$\frac{dW}{dt} = P - P_{\text{loss}} - F \frac{dx}{dt} \quad (31)$$

where $F(dx/dt)$ is the rate of work done by the radiation pressure on the end disk. By expressing the power loss P_{loss} again in terms of $(\omega/Q)W$, where W is the stored energy, equation (31) becomes

$$\frac{dW}{dt} = P - \frac{\omega_g}{Q} W - F \frac{dx}{dt} \quad (32)$$

By writing $W = Fx = (W/x)x$, dW/dt can be expressed as

$$\frac{dW}{dt} = F \frac{dx}{dt} + x \frac{dF}{dt}$$

Substituting dW/dt into equation (32) gives

$$\frac{dF}{dt} = \frac{P}{x} - \frac{\omega_g}{Q} F - \frac{2F}{x} \frac{dx}{dt} \quad (33)$$

For purposes of physical insight, the terms in equation (33) are separately interpreted. Assume that energy is being stored in the cavity before the disk is moving (the drag of a stationary magnetic field through which the plasma has to move could serve the purpose) and that the energy is used to accelerate the plasma without further power input P and power loss P_{loss} of $\omega_g W/Q$. The change in work done at the expense of change in stored energy is then $-F dx$; thus,

$$dW = F dx + x dF = -F dx \quad (34)$$

It follows that

$$\frac{dF}{F} = -2 \frac{dx}{x} \quad (35)$$

Integration of equation (35) yields

$$\frac{F}{F_0} = \left(\frac{x_0}{x} \right)^2 \quad (36)$$

This law of adiabatic expansion should be compared with that for the adiabatic expansion of black-body radiation (ref. 16) where the exponent γ is equal to $4/3$ instead of to the present value of 2. The reason for the difference can be seen from the definition of γ , which is the ratio of specific heats, in terms of degrees of freedom n' :

$$\gamma = \frac{n' + 2}{n'} \quad (37)$$

An arbitrary wave traveling in space can be composed of plane waves traveling in the direction of the Cartesian coordinate axes. Each plane wave has two degrees of freedom. For the present case of a wave traveling along the axis of the tube, $n' = 2$ and

$$\gamma = \frac{2 + 2}{2} = 2 \quad (38)$$

For arbitrary waves used in the analysis of black-body radiation, $n' = 3 \times 2 = 6$ or

$$\gamma = \frac{6 + 2}{6} = \frac{4}{3} \quad (39)$$

For the present preliminary calculations, the acceleration of the plasma is evaluated from the simplified adiabatic expansion law (see eq. (36)) which is rewritten as

$$F = m \frac{d^2x}{dt^2} = \frac{F_0 x_0^2}{x^2} \quad (40)$$

In other words, the term P/x related to the power added during the acceleration and the loss term $\frac{\omega_g}{Q} F$ in equation (33) are assumed to be the same. The following estimate suggests that the aforementioned assumption is rather conservative: The estimate is based in essence on the first step of an iterative procedure using for F the adiabatic relation $F = F_0 (x_0/x)^2$ in the evaluation of the term $(\omega_g/Q)F$, whereby the expression for $F_{\max} = \frac{P}{x_0} \frac{Q}{\omega_{g,o}}$ (eq. (27)) is substituted for F_0 ; thus,

$$\frac{\omega_g}{Q} F = \frac{\omega_g}{Q} \frac{P Q_0}{\omega_{g,o} x_0} \frac{x_0^2}{x^2} = \frac{P}{x} \frac{x_0}{x} \frac{\omega_g}{\omega_{g,o}} \frac{Q_0}{Q}$$

where the subscript o refers to conditions for the initial nonexpanded cavity. During the expansion of the radiation, the wavelength λ_g has to be increased to maintain resonance conditions and with it the resonant frequency decreases. The initial frequency is, however, so far from the cutoff frequency that the losses should not increase too much. The resulting reduction in Q or $Q_0/Q > 1$ would not seem so large as to overcome the effect of $x_0/x < 1$ or $\omega_g/\omega_{g,o} < 1$ during the expansion. The loss term $(\omega_g/Q)F$ should thus remain below the input term P/x during the expansion, a condition making the adiabatic expansion conservative. For a complete solution, $F = m \frac{d^2x}{dt^2}$ must be substituted into equation (33), and the result is

$$m \frac{d^3x}{dt^3} = \frac{P}{x} - \frac{\omega_g}{Q} \frac{d^2x}{dt^2} - \frac{2m}{x} \frac{d^2x}{dt^2} \frac{dx}{dt} \quad (41)$$

Thus, a third-order equation has to be solved. The solution of equation (41) is, however, not discussed here.

Velocity Attainable by Plasma

The velocity is obtained by integration of the simplified equation (40) which gives

$$\ddot{x} = \frac{F_0 x_0^2}{m} \frac{1}{x^2}$$

where a dot over a symbol indicates a time derivative. Multiplying \ddot{x} by $2\dot{x}$ gives

$$2\dot{x}\ddot{x} = \frac{d}{dt}(\dot{x}^2) = 2 \frac{F_0 x_0^2}{m} \frac{\dot{x}}{x^2} = -\frac{F_0 x_0^2}{m} \frac{d}{dt}\left(\frac{2}{x}\right) \quad (42)$$

Integration of equation (42) gives

$$\dot{x}^2 = -\frac{F_0 x_0^2}{m} \frac{2}{x} + C' \quad (43)$$

The acceleration starts at a distance x_0 (the initial cavity length) at zero velocity, $\dot{x} = 0$; thus, energy is assumed to be stored in the cavity before the plasma disk is moving. Therefore,

$$C' = \frac{2F_0 x_0}{m} \quad (44)$$

and

$$\dot{x}^2 = \frac{2F_0 x_0}{m} \left(1 - \frac{x_0}{x}\right) \quad (45)$$

or

$$\dot{x} = \sqrt{\frac{2F_0 x_0}{m} \left(1 - \frac{x_0}{x}\right)} \quad (46)$$

For the present approximate calculations, the force F_0 at the cavity end disk is given by the simple expression for F_{\max} from equation (27)

$$F_0 = F_{\max} = \frac{P}{x_0} \frac{Q}{\omega_{g,0}}$$

where the length in Q corresponds to the initial cavity length x_0 . The result is

$$\dot{x} = \sqrt{\frac{2PQ}{m\omega_{g,0}} \left(1 - \frac{x_0}{x}\right)} \quad (47)$$

Numerical estimates of the velocities \dot{x} attainable require an estimate of the plasma mass m . In agreement with the previous discussion, it is assumed that the mass consists of 10^{14} particles of hydrogen where the mass of a hydrogen atom is about 1.6×10^{-24} gram (or twice as high for deuterium). Assume that the plasma disk has a cross-sectional area of 100 square centimeters and a thickness of 1 centimeter. The thickness is about 10 times the penetration depth for total dielectric reflection from the plasma. The resulting mass of the plasma would be 1.6×10^{-8} gram or 1.6×10^{-11} kilogram. Since the TE_0 mode has its maximum strength in a ring-shaped region, the plasma volume could be differently distributed in the cross section of the guide.

It is further necessary to choose values for x_0/x , the ratio of initial cavity length x_0 to the position x of the plasma during its motion. Since frequency changes up to 20 percent are becoming possible for existing traveling wave-tube transmitters and since the wavelength is about proportional to x (if about one-dimensional expansion of the radiation is assumed), one value of the ratio x_0/x tried is 10/12. By assuming that several transmitters could be phased, larger frequency changes would seem possible; as an example, the velocities are also calculated for $x_0/x = 1/2$ which corresponds to doubling of the wavelength during the acceleration. Note that, according to equation (46), the velocity \dot{x} could, however, be also increased by an increase in the initial length x_0 of the cavity. Since in the present approximate approach the frequency $\omega_{g,0}$ enters directly only through the constant F_0 , $\omega_{g,0}$ is fixed at $2\pi f_{g,0}$ where $f_0 = 10^{10}$ cycles/sec; the power

is 10^8 watts, as previously used. The velocities attainable for a plasma conductivity of $\sigma = 10^3$ mhos/m and for $x_0/x = 10/12$ are plotted in figure 7 for the cases of ordinary and superconducting sidewalls. The velocities for $x_0/x = 1/2$ and $\sigma = 10^3$ mhos/m are given in figure 8. The values for the velocities are conservative since only one wall of the metallic cavity has the comparatively low plasma conductivity of $\sigma = 10^3$ mhos/m. Similar calculations for $\sigma = 6.13 \times 10^7$ mhos/m are presented in figures 9 and 10. It should be noted that, according to these approximate calculations, plasma velocities of 10^5 m/sec would not seem too hard to obtain, even for plasma conductivities of 10^3 mhos/m. For extreme plasma conductivities of 6.13×10^7 mhos/m, velocities of the order of even 10^7 m/sec result (without relativistic corrections). In all these calculations it has been assumed that the plasma is accelerated into a vacuum so that its mass will not increase during the acceleration.

It is again of interest to make a comparison with the nonresonant case. In that case an approximately constant force of $1/3$ or $2/3$ newton is applied since the wave train makes only one passage (or two with a single reflection) and the sidewall losses are negligible (as in the case of superconducting sidewalls).

The differential equation of motion is now

$$m\ddot{x} = F = \text{Constant} \quad (48)$$

and can be solved easily without approximations. By assuming that the initial velocity $\dot{x} = 0$ and the plasma motion start at $x = 0$, the result for \dot{x} is

$$\dot{x} = \sqrt{\frac{2Fx}{m}} \quad (49)$$

For a force $F = 1/3$ newton and a distance $x = 1\text{m}$ of plasma motion using a mass of 1.6×10^{-11} kilogram, the result is

$$\dot{x} \approx 2 \times 10^5 \text{ m/sec}$$

This result is lower on the average than the velocity attainable for the resonant case with $\sigma = 10^3$ mhos/m given in figures 7 and 8. The

resonant case for $\sigma = 6.13 \times 10^7$ mhos/m (figs. 9 and 10) yields much higher values. Recall that the force F_0 for the resonant case was more than 10 times larger and the force enters under the square-root sign in the velocity. Although a considerable increase in velocity could probably be attained (a factor of 5 or larger already counts much in the velocity scale) for the resonant case with $\sigma = 10^3$ mhos/m, it is significant to note that even without resonance considerable velocities can be attained. Note that for the nonresonant case no frequency variation is required from the receiver since no expansion of the radiation takes place. It could be thus conceived that the initial large push over a comparatively small frequency range could be provided by the resonance method but that the push should be continued in a nonresonant manner over a greater length. It is conceivable that this nonresonant push could be continued over a large distance if a large number of high-powered generators are coupled in from the sides of the waveguide. In this manner, matching problems which arise in other accelerators could be avoided throughout the acceleration. Note that the "matching" problem for the initial resonant case with frequency variation seems to be solvable with existing feedback methods unless the plasma velocities are to approach a large fraction of the speed of light, but at these extreme velocities the nonresonant push can take over.

A brief discussion of the feasibility of simulating micrometeorites with the present method may also be of interest. For the TE_0 mode which has currents flowing through circles in the waveguide cross section, the micrometeorites could be simulated by accelerating connected concentric wire rings. The wires which would have the diameter d of the micrometeorite, say $d = 4 \times 10^{-3}$ centimeter, could be broken up after acceleration. In this manner the masses to be accelerated could be made extremely small, especially if aluminum is used which still has very good conductivity but a specific weight of only 2.6 g/cm^3 . For waveguides with a 6-centimeter radius, which were previously discussed, the masses would be on the order of 10^{-6} kilogram. The velocities attainable are reduced compared with those of the plasma in the inverse ratio of the square root of their masses; that is, multiplication of the values of the velocity in figures 9 or 10 indicates that the attainment of meteorite velocities of approximately 40 km/sec is within the range of possibility. This is especially true if an extra push is provided in a nonresonant manner. Several methods for reducing the heating of the aluminum ring are being considered.

CONCLUDING REMARKS

With the assumption of a power input of 10^8 watts into a cavity, an input which lies in the range of possibilities because of the recent development of modern generators, forces on the order of 1,000 newtons seem to be attainable for highly conducting thin plasma disks or rings. For such plasmas, velocities up to 10^6 m/sec and more are calculated. Although the energies that can be stored in the cavity do not yet quite measure up to acceleration systems where the energy is stored in condenser banks, the efficiency of interaction of the radiation with the plasma could be better than that of other systems. In that manner it could be possible to accelerate plasmas of small masses to higher velocities than in other systems. Since the plasma proves to be a very good reflector at high temperatures, special designs of plasma accelerators have also been considered where both end walls of the cavity are made of plasmas with arrangements enabling alternate replacement of the plasmas after acceleration. In this manner, the power generally lost due to skin effect could be put to use. The acceleration of micrometeorites with this method is also found to be within the range of possibilities.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., June 29, 1959.

REFERENCES

1. Patrick, R. M.: A Description of a Propulsive Device Which Employs a Magnetic Field as the Driving Force. Res. Rep. 28 (AFOSR TN 58-684, ASTIA AD 162 217), AVCO Res. Lab., May 1958.
2. Thonemann, P. C., Cowhig, W. T., and Davenport, P. A.: Interaction of Travelling Magnetic Fields With Ionized Gases. Nature (Letters to the Editors), vol. 169, no. 4288, Jan. 5, 1952, pp. 34-35.
3. Marshall, John: Acceleration of Plasma into Vacuum. Proc. Second United Nations Int. Conf. on Peaceful Uses of Atomic Energy (Geneva), vol. 31 - Theoretical and Experimental Aspects of Controlled Nuclear Fusion, 1958, pp. 341-347.
4. Butler, J. W., Hatch, A. J., and Ulrich, A. J.: Radio-Frequency Thermonuclear Machines. Proc. Second United Nations Int. Conf. on Peaceful Uses of Atomic Energy (Geneva), vol. 32 - Controlled Fusion Devices, 1958, pp. 324-332.
5. Clauser, Milton U., and Weibel, E. S.: Radiation Pressure Confinement, the Shock Pinch and Feasibility of Fusion Propulsion. Proc. Second United Nations Int. Conf. on Peaceful Uses of Atomic Energy (Geneva), vol. 32 - Controlled Fusion Devices, 1958, pp. 161-168.
6. Vedenov, A. A., and Volkov, T. F., et al.: Thermal Insulation and Confinement of Plasma With a High-Frequency Electromagnetic Field. Proc. Second United Nations Int. Conf. on Peaceful Uses of Atomic Energy (Geneva), vol. 32 - Controlled Fusion Devices, 1958, pp. 239-244.
7. Livingston, M. Stanley: High-Energy Accelerators. Interscience Publ., Inc. (New York), 1954, p. 96.
8. Skilling, Hugh Hildreth: Fundamentals of Electric Waves. Second ed., John Wiley & Sons, Inc., 1948.
9. Sarbacher, Robert I., and Edson, William A.: Hyper and Ultrahigh Frequency Engineering. John Wiley & Sons, Inc., 1944, p. 265.
10. Reintjes, J. Francis, and Coate, Godfrey T.: Principles of Radar. Third ed., McGraw-Hill Book Co., Inc., 1952, p. 610.
11. Spitzer, Lyman, Jr.: Physics of Fully Ionized Gases. Interscience Publ., Inc. (New York), 1956.
12. Montgomery, Carol G., ed.: Technique of Microwave Measurements. McGraw-Hill Book Co., Inc., 1947.

13. Slater, John C., and Frank, Nathaniel H.: Electromagnetism. McGraw-Hill Book Co., Inc., 1947.
14. Pohl, Robert Wichard: Elektrizitätslehre. Springer-Verlag (Berlin), 1955, p. 308.
15. Dreicer, H.: Theory of Runaway Electrons. Proc. Second United Nations Int. Conf. on Peaceful Uses of Atomic Energy (Geneva), vol. 31 - Theoretical and Experimental Aspects of Controlled Nuclear Fusion, 1958, pp. 57-64.
16. Richtmyer, F. K., Kennard, E. H., and Lauritsen, T.: Introduction to Modern Physics. Fifth ed., McGraw-Hill Book Co., Inc., 1955, p. 115.

TABLE I

VALUES OF Q DEPENDING ON LENGTH OF CAVITY

$$(a) \quad x_0 = \frac{\lambda_g}{2} \text{ s; } \sigma = 10^3 \text{ mhos/m}$$

| s | Q | Q_{super} |
|----------|----------------------|----------------------|
| 1 | 55.74 | 56.55 |
| 10 | 496.79 | 565.45 |
| 20 | 888.57 | 1.1309×10^3 |
| 30 | 1.2036×10^3 | 1.6964×10^3 |
| 40 | 1.4661×10^3 | 2.2618×10^3 |
| 50 | 1.6842×10^3 | 2.8273×10^3 |
| 60 | 1.8579×10^3 | 3.3927×10^3 |
| 70 | 2.0316×10^3 | 3.9582×10^3 |
| 80 | 2.1689×10^3 | 4.5236×10^3 |
| 90 | 2.2901×10^3 | 5.0891×10^3 |
| 100 | 2.3991×10^3 | 5.6545×10^3 |
| ∞ | 4.287×10^3 | ∞ |

TABLE I.- Concluded

VALUES OF Q DEPENDING ON LENGTH OF CAVITY

$$(b) \quad x_0 = \frac{\lambda g}{2} s; \quad \sigma = 6.13 \times 10^7 \text{ mhos/m}$$

| s | Q | Q_{super} |
|-----|--------------------|--------------------|
| 1 | 1.38×10^4 | 1.4×10^4 |
| 10 | 1.23×10^5 | 1.4×10^4 |
| 20 | 2.20×10^5 | 2.8×10^5 |
| 30 | 2.98×10^5 | 4.2×10^5 |
| 40 | 3.63×10^5 | 5.6×10^5 |
| 50 | 4.17×10^5 | 7.0×10^5 |
| 60 | 4.60×10^5 | 8.4×10^5 |
| 70 | 5.03×10^5 | 9.8×10^5 |
| 80 | 5.37×10^5 | 1.12×10^6 |
| 90 | 5.67×10^5 | 1.26×10^6 |
| 100 | 5.94×10^5 | 1.4×10^6 |

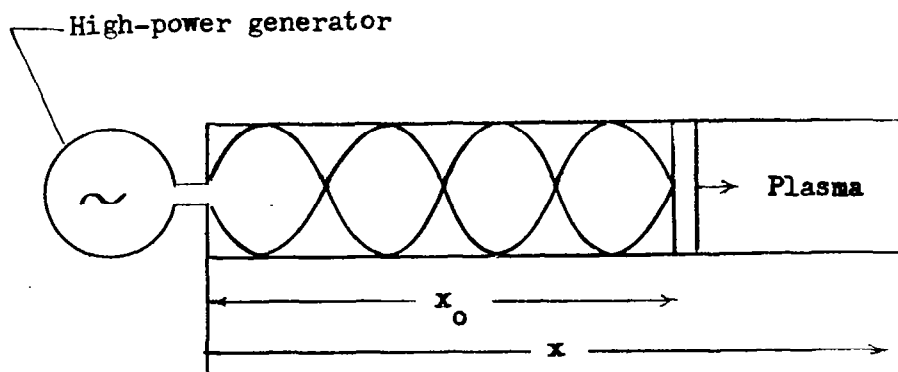


Figure 1.- Resonance in cavity.

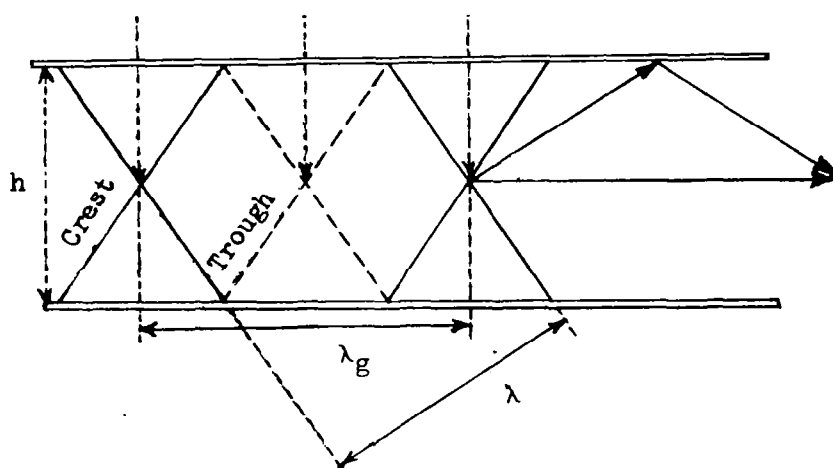


Figure 2.- Waveguide relations.

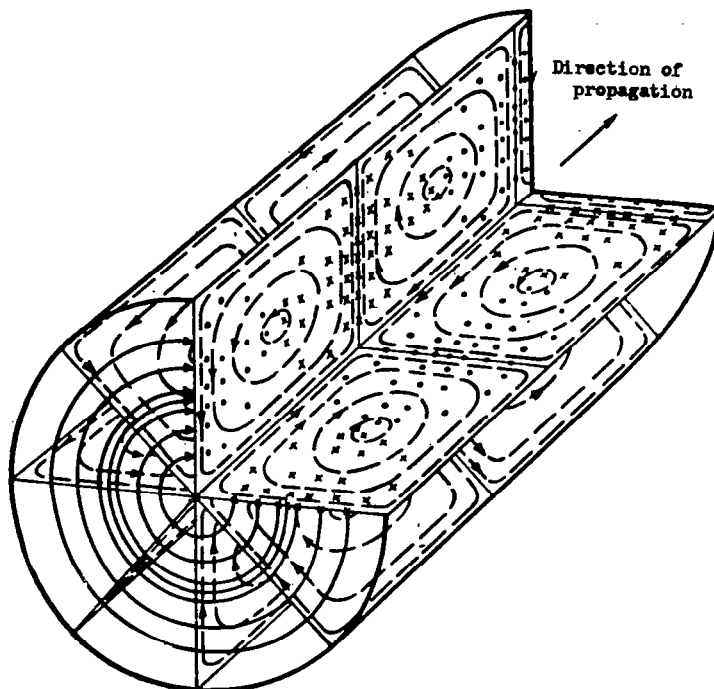


Figure 3.- The $TE_{0,1}$ mode as shown in reference 10.

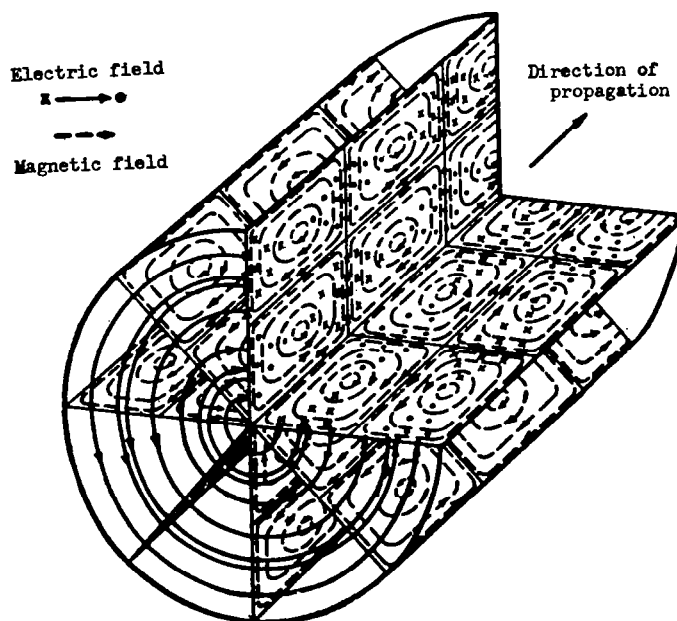


Figure 4.- The $TE_{0,2}$ mode as shown in reference 10.

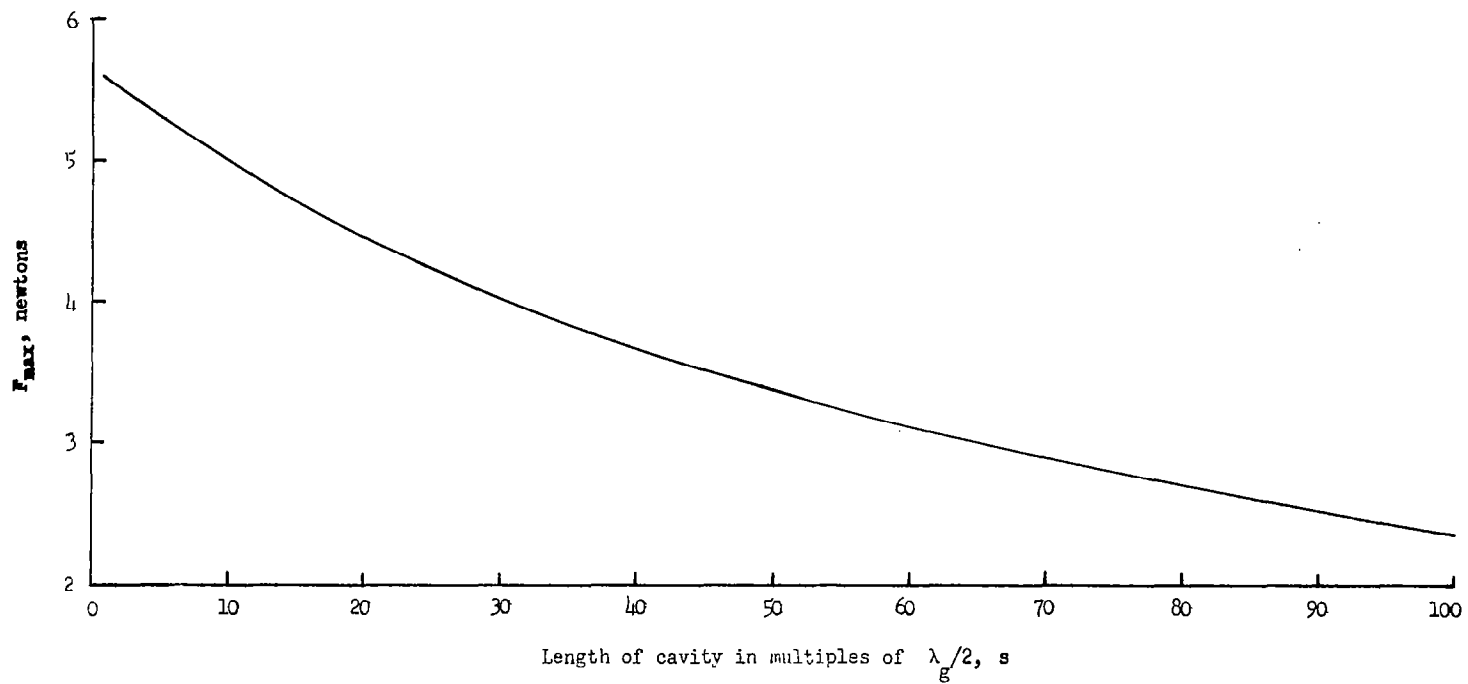


Figure 5.- Force depending on length of cavity x_0 with $\sigma = 10^3$ mhos/m.

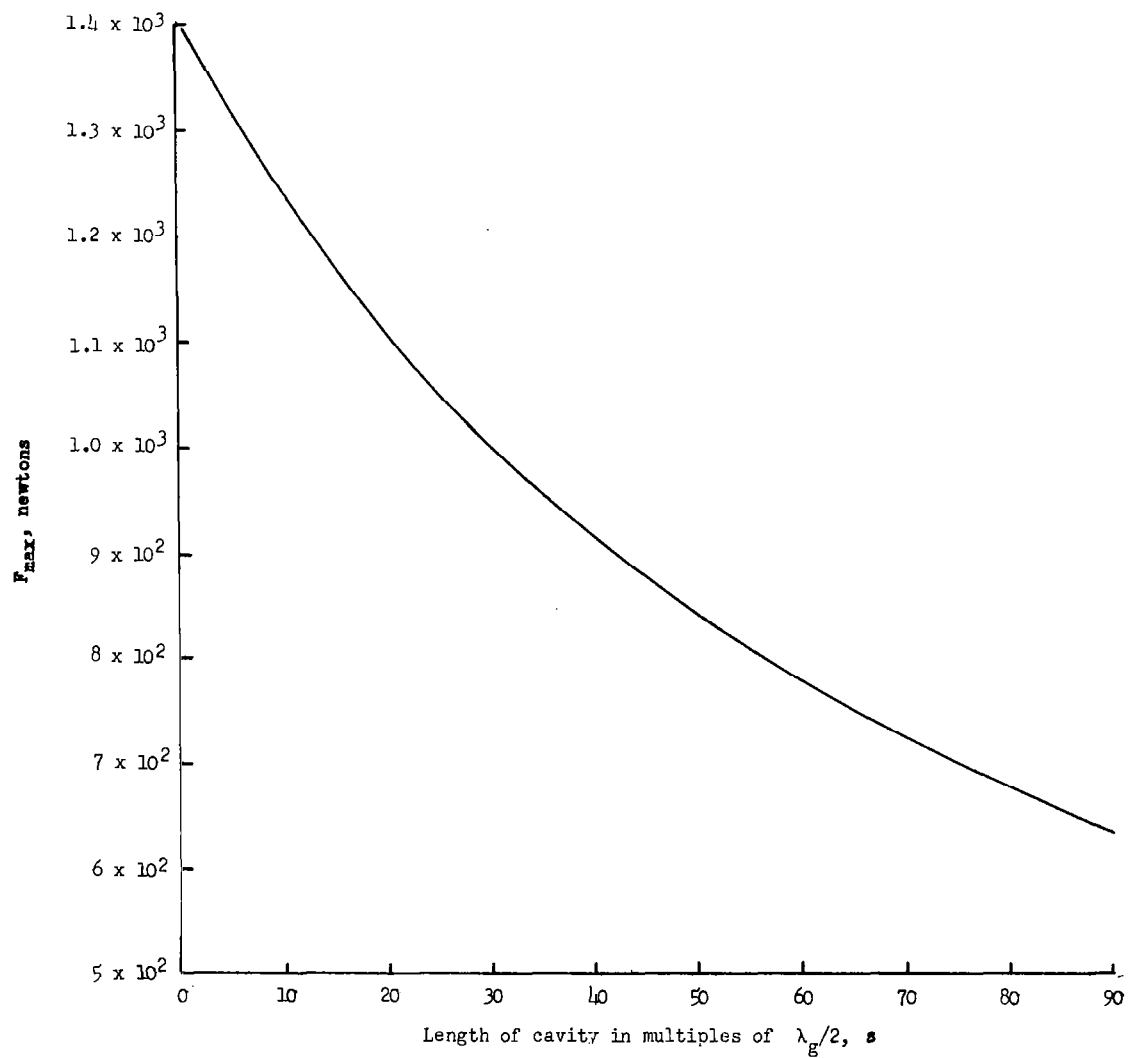


Figure 6.- Force depending on length of cavity x_0 with $\sigma = 6.13 \times 10^7$ mhos/m.

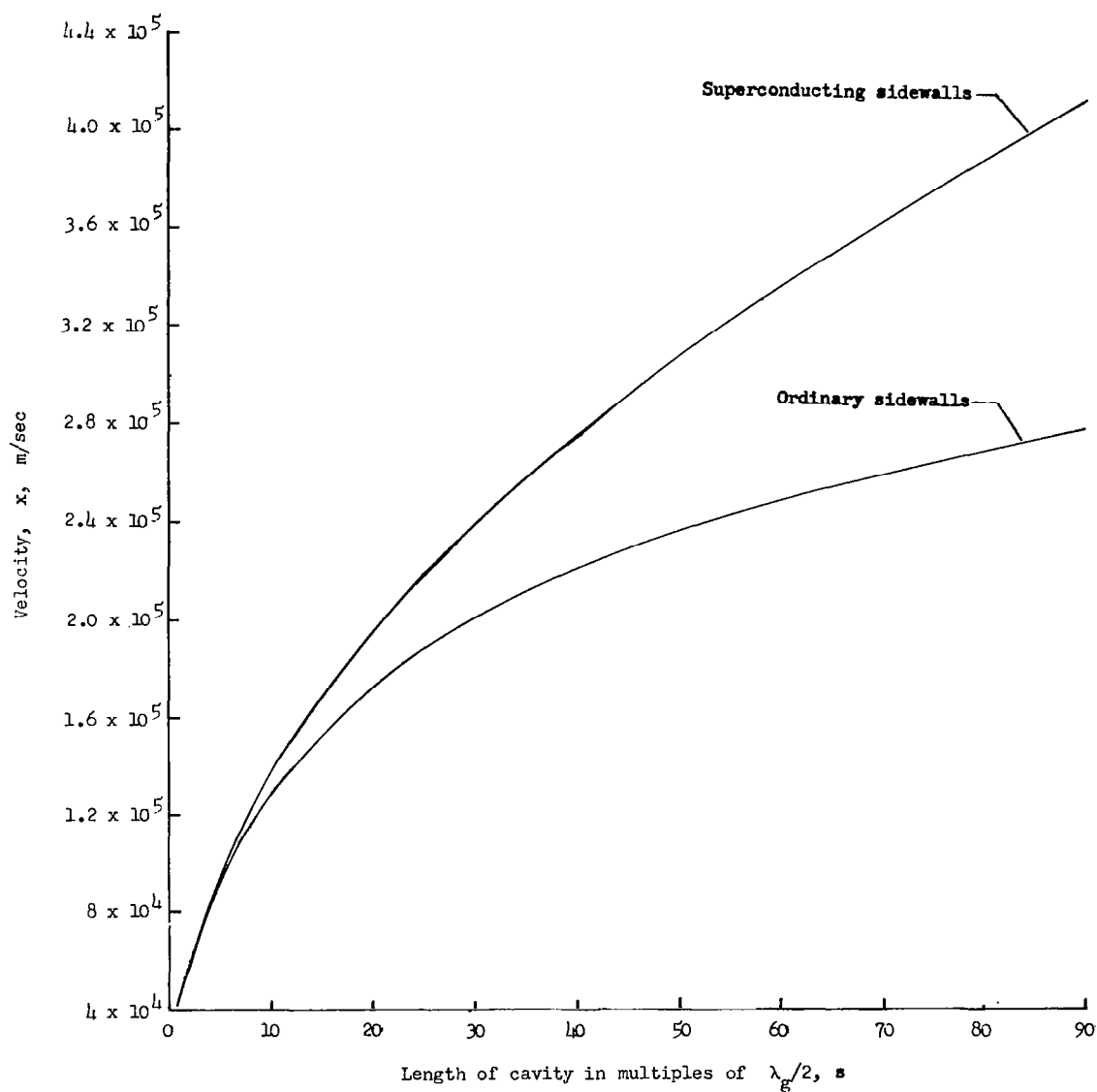


Figure 7.- Velocity \dot{x} depending on length of cavity x_0 with $\sigma = 10^3$ mhos/m and $x_0/x = 5/6$.

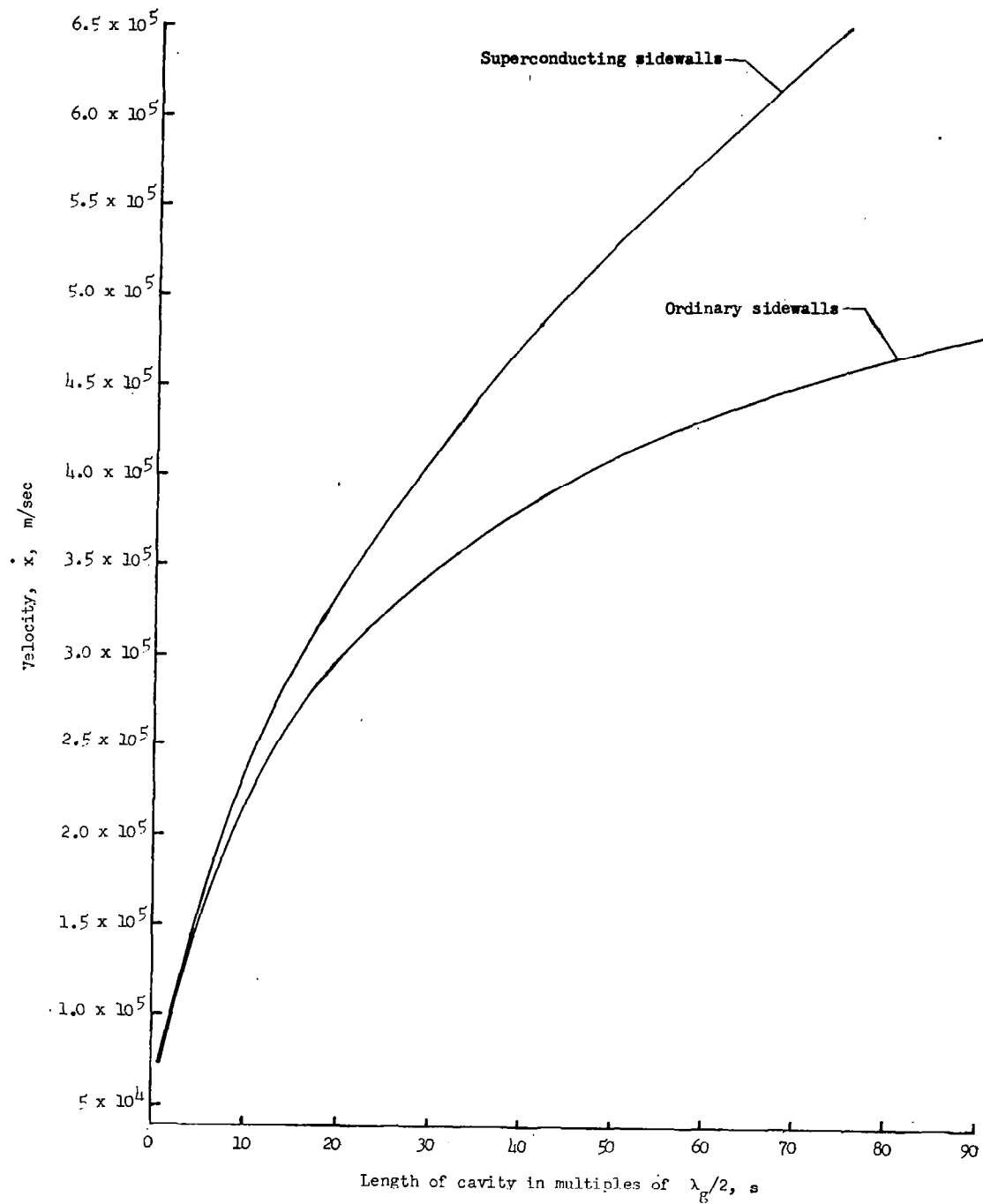


Figure 8.- Velocity \dot{x} depending on length of cavity x_0 with $\sigma = 10^3$ mhos/m and $x_0/x = 1/2$.

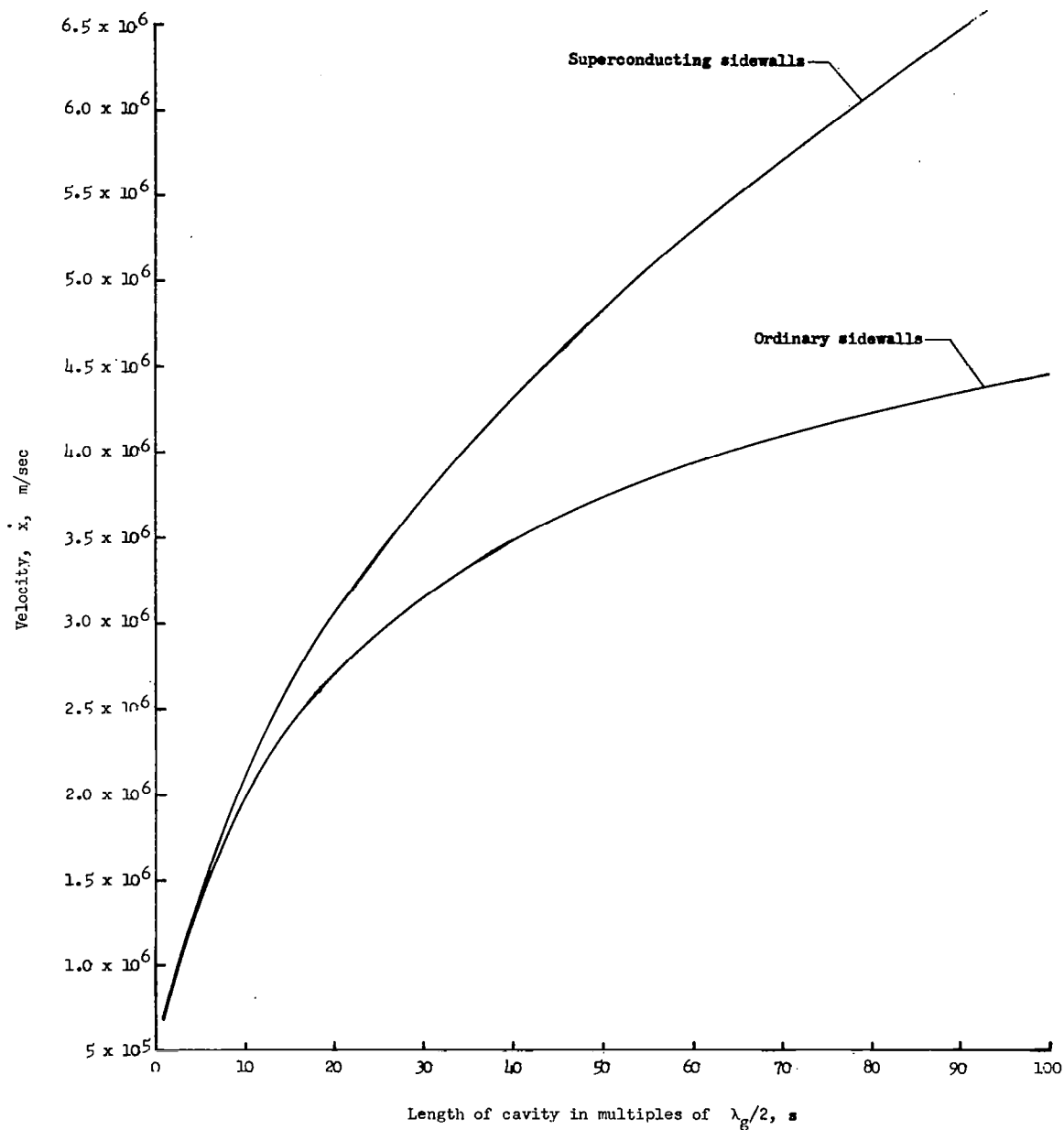


Figure 9.- Velocity \dot{x} depending on length of cavity x_0 with $\sigma = 6.13 \times 10^7$ mhos/m and $x_0/x = 5/6$.

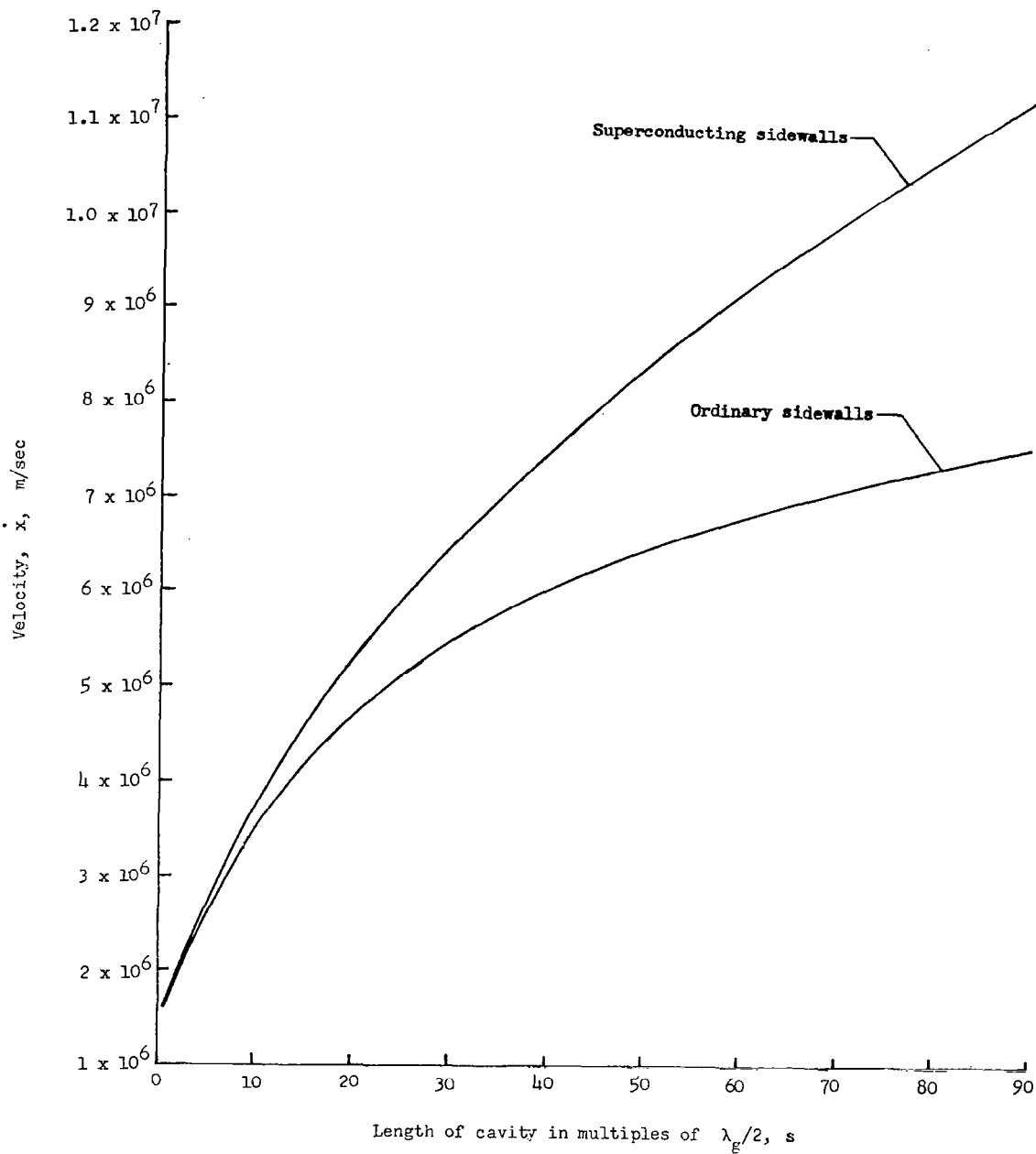


Figure 10.- Velocity \dot{x} depending on length of cavity x_0 with $\sigma = 6.13 \times 10^7$ mhos/m and $x_0/x = 1/2$.

COMMANDING GENERAL
AIR FORCE SPECIAL WEAPONS CENTER
ATT. SWOI
KIRPLAND AIR FORCE BASE, NEW MEXICO

T. N. D-46

1-4, 7-11, 16, 17, 20, 21, 23-25, 27, 28,
30-32, 34, 36, 37, 40-41, 43, 49, 52.